

Charm CP Asymmetry Sum Rules from SU(3)-Flavor

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based on Müller, Nierste, StS: arXiv:1503.06759 and in preparation

Can we resolve new physics in charm decays?

Goal: Get the most out of \mathcal{B} 's in order to predict CP asymmetries.

Red: Update in 2014

Observable	Measurement
SCS CP asymmetries	
$\Delta a_{CP}^{\text{dir}}(K^+ K^-, \pi^+ \pi^-)$	-0.00253 ± 0.00104
$\Sigma a_{CP}^{\text{dir}}(K^+ K^-, \pi^+ \pi^-)$	-0.0011 ± 0.0026
$a_{CP}^{\text{dir}}(D^0 \rightarrow K_S K_S)$	-0.23 ± 0.19
$a_{CP}^{\text{dir}}(D^0 \rightarrow \pi^0 \pi^0)$	-0.0004 ± 0.0064
$a_{CP}^{\text{dir}}(D^+ \rightarrow \pi^0 \pi^+)$	$+0.029 \pm 0.029$
$a_{CP}^{\text{dir}}(D^+ \rightarrow K_S K^+)$	$+0.0011 \pm 0.0017$
$a_{CP}^{\text{dir}}(D_s \rightarrow K_S \pi^+)$	$+0.006 \pm 0.005$
$a_{CP}^{\text{dir}}(D_s \rightarrow K^+ \pi^0)$	$+0.266 \pm 0.228$
Indirect CP violation	
a_{CP}^{ind}	0.00013 ± 0.00052
$\delta_L \equiv 2\text{Re}(\varepsilon)/(1 + \varepsilon ^2)$	$(3.32 \pm 0.06) \cdot 10^{-3}$
$K^+ \pi^-$ strong phase difference	
$\delta_{K\pi}$	$(6.45 \pm 10.65)^\circ$

Observable	Measurement
SCS branching ratios	
$\mathcal{B}(D^0 \rightarrow K^+ K^-)$	$(3.96 \pm 0.08) \cdot 10^{-3}$
$\mathcal{B}(D^0 \rightarrow \pi^+ \pi^-)$	$(1.402 \pm 0.026) \cdot 10^{-3}$
$\mathcal{B}(D^0 \rightarrow K_S K_S)$	$(0.17 \pm 0.04) \cdot 10^{-3}$
$\mathcal{B}(D^0 \rightarrow \pi^0 \pi^0)$	$(0.820 \pm 0.035) \cdot 10^{-3}$
$\mathcal{B}(D^+ \rightarrow \pi^0 \pi^+)$	$(1.19 \pm 0.06) \cdot 10^{-3}$
$\mathcal{B}(D^+ \rightarrow K_S K^+)$	$(2.83 \pm 0.16) \cdot 10^{-3}$
$\mathcal{B}(D_s \rightarrow K_S \pi^+)$	$(1.22 \pm 0.06) \cdot 10^{-3}$
$\mathcal{B}(D_s \rightarrow K^+ \pi^0)$	$(0.63 \pm 0.21) \cdot 10^{-3}$
CF branching ratios	
$\mathcal{B}(D^0 \rightarrow K^- \pi^+)$	$(3.88 \pm 0.05) \cdot 10^{-2}$
$\mathcal{B}(D^0 \rightarrow K_S \pi^0)$	$(1.19 \pm 0.04) \cdot 10^{-2}$
$\mathcal{B}(D^0 \rightarrow K_L \pi^0)$	$(1.00 \pm 0.07) \cdot 10^{-2}$
$\mathcal{B}(D^+ \rightarrow K_S \pi^+)$	$(1.47 \pm 0.07) \cdot 10^{-2}$
$\mathcal{B}(D^+ \rightarrow K_L \pi^+)$	$(1.46 \pm 0.05) \cdot 10^{-2}$
DCS branching ratios	
$\mathcal{B}(D^0 \rightarrow K^+ \pi^-)$	$(1.35 \pm 0.02) \cdot 10^{-4}$
$\mathcal{B}(D^+ \rightarrow K^+ \pi^0)$	$(1.83 \pm 0.26) \cdot 10^{-4}$

The Problem



Charm is **not really heavy** compared to Λ_{QCD}

- $m_c \sim 1.3 \text{ GeV}$, $m_b \sim 4.2 \text{ GeV}$.
- Perturbative expansion in Λ_{QCD}/m_c will **not** work.

Available tools

- SU(3)_F expansion.
- Topological amplitudes.
- $1/N_c$ expansion.

[t Hooft 1974, Buras Gerard Rückl
1986]



➡ The best we can do at present.

Calculate what is calculable. Fit all other hadronic parameters from \mathcal{B} 's.

't Hooft 1974: Study $SU(3)_C \Rightarrow SU(N_c)_C$

N_c = number of colors.

- Asymptotic freedom \Rightarrow Expansion in $\alpha_s(\mu)$ works for high energies.
- Breaks down for low energy QCD \Rightarrow Nonperturbative regime.
- Consider $N_c \rightarrow \infty$ and expand in $1/N_c$.
- $g = g_0 / \sqrt{N_c}$, $g^2 \sim 1/N_c$.

gluon vertex:



$\mathcal{O}(1/\sqrt{N_c})$

closed loop:



$\mathcal{O}(N_c)$

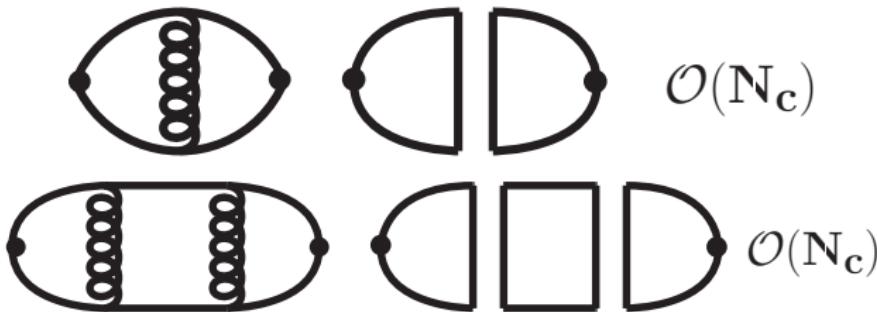
meson vertex:



$\mathcal{O}(1/\sqrt{N_c})$

$1/N_c$ power counting

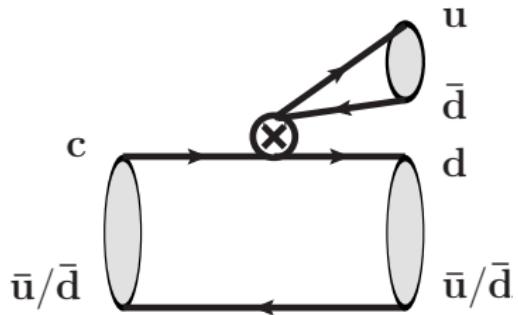
Corrections of the **same order**:



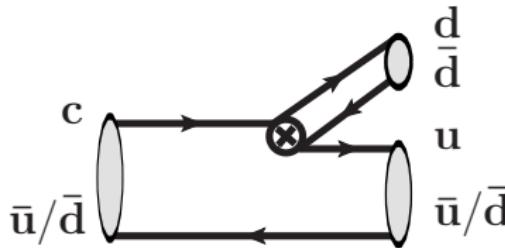
Suppressed corrections:



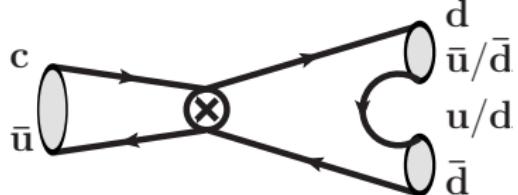
Application to charm decay topologies



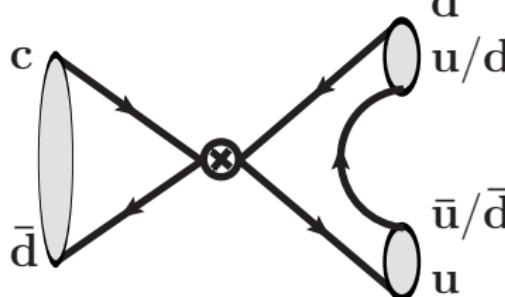
tree (T)



color-suppressed tree (C)



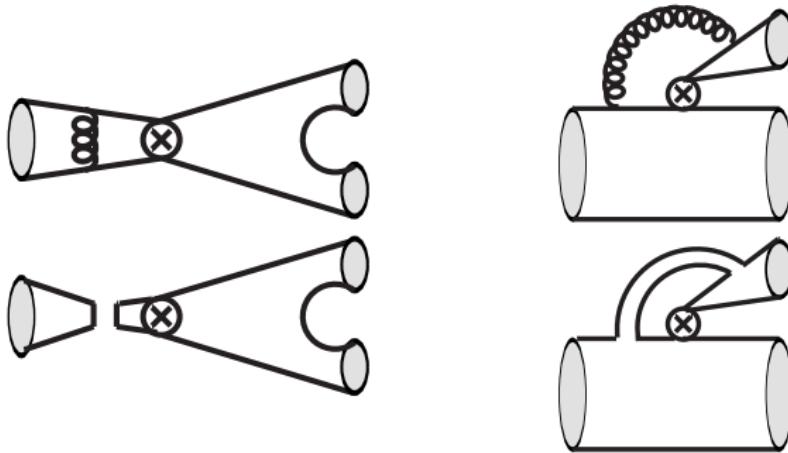
exchange (E)



annihilation (A)

[Chau 1980,1982; Zeppenfeld 1981, Buras Silvestrini 1998]

Corrections to T and A diagrams $1/N_c^2$ suppressed



same order in $1/N_c$ $1/N_c^2$ -suppressed.
 \Rightarrow fit E . \Rightarrow fit $\delta_T \leq 15\%$ in $T = T^{\text{fac}}(1 + \delta_T)$,
 analogous: \Rightarrow fit $\delta_A \leq 15\%$ in $A = A^{\text{fac}}(1 + \delta_A)$

for example:

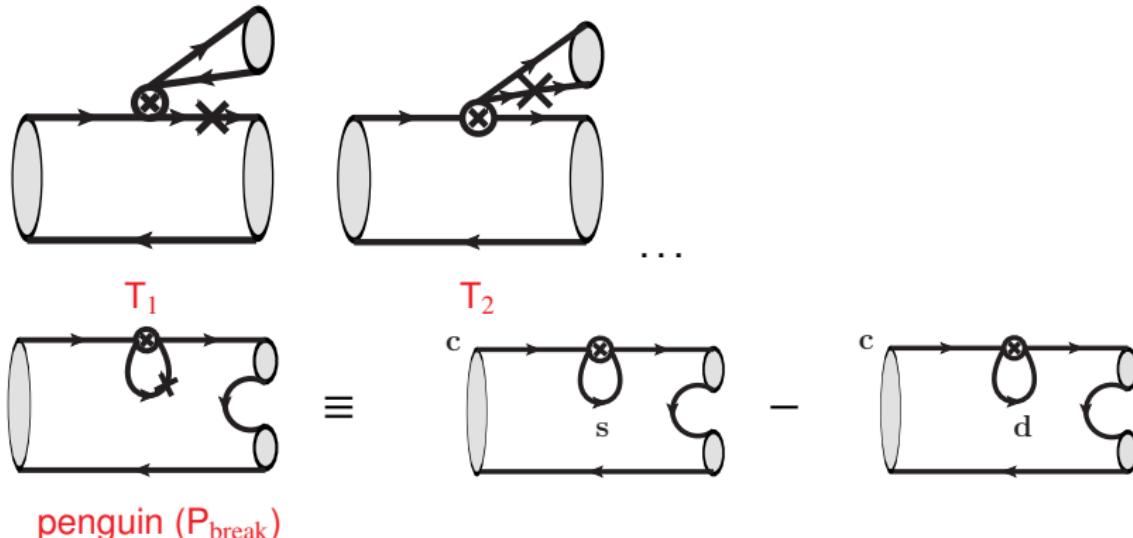
$$T(D^0 \rightarrow K^+ K^-) = \frac{G_F}{\sqrt{2}} a_1 f_K (m_D^2 - m_K^2) F^{DK}(m_K^2) \left(1 + O(1/N_c^2)\right)$$

$$A(D_s^+ \rightarrow K^0 \pi^+) = \frac{G_F}{\sqrt{2}} a_1 f_{D_s} (m_K^2 - m_\pi^2) F^{K\pi}(m_{D_s}^2) \left(1 + O(1/N_c^2)\right)$$

Diagrammatic $SU(3)_F$ breaking

- Feynman rule from $H_{SU(3)_F} = (m_s - m_d)\bar{s}s$: dot on s -quark line. [Gronau 1995]
- Find 14 new topological amplitudes:
3 diagrams for each T, C, E, A ; $P_{\text{break}} \equiv P_d - P_s$; $PA_{\text{break}} \equiv PA_d - PA_s$.

[Brod Grossman Kagan Zupan 2012]



Diagrammatic Parameterization (excerpt)

Decay d	T	$T_1^{(1)}$	$T_2^{(1)}$	$T_3^{(1)}$	A	$A_1^{(1)}$	$A_2^{(1)}$	$A_3^{(1)}$	C	$C_1^{(1)}$	$C_2^{(1)}$	$C_3^{(1)}$...
SCS													
$D^0 \rightarrow K^+ K^-$	1	1	1	0	0	0	0	0	0	0	0	0	...
$D^0 \rightarrow \pi^+ \pi^-$	-1	0	0	0	0	0	0	0	0	0	0	0	...
$D^0 \rightarrow \bar{K}^0 K^0$	0	0	0	0	0	0	0	0	0	0	0	0	...
$D^0 \rightarrow \pi^0 \pi^0$	0	0	0	0	0	0	0	0	$-\frac{1}{\sqrt{2}}$	0	0	0	...
$D^+ \rightarrow \pi^0 \pi^+$	$-\frac{1}{\sqrt{2}}$	0	0	0	0	0	0	0	$-\frac{1}{\sqrt{2}}$	0	0	0	...
$D^+ \rightarrow \bar{K}^0 K^+$	1	1	1	0	-1	0	0	-1	0	0	0	0	...
$D_s \rightarrow K^0 \pi^+$	-1	0	0	-1	1	1	1	0	0	0	0	0	...
$D_s \rightarrow K^+ \pi^0$	0	0	0	0	$-\frac{1}{\sqrt{2}}$	$-\frac{1}{\sqrt{2}}$	$-\frac{1}{\sqrt{2}}$	0	$-\frac{1}{\sqrt{2}}$	0	0	$-\frac{1}{\sqrt{2}}$...
CF													
$D^0 \rightarrow K^- \pi^+$	1	1	0	0	0	0	0	0	0	0	0	0	...
$D^0 \rightarrow \bar{K}^0 \pi^0$	0	0	0	0	0	0	0	0	$\frac{1}{\sqrt{2}}$	$\frac{1}{\sqrt{2}}$	0	0	...
$D^+ \rightarrow \bar{K}^0 \pi^+$	1	1	0	0	0	0	0	0	1	1	0	0	...
$D_s \rightarrow \bar{K}^0 K^+$	0	0	0	0	1	1	0	1	1	1	0	1	...
DCS													
$D^0 \rightarrow K^+ \pi^-$	1	0	1	0	0	0	0	0	0	0	0	0	...
$D^0 \rightarrow K^0 \pi^0$	0	0	0	0	0	0	0	0	$\frac{1}{\sqrt{2}}$	0	$\frac{1}{\sqrt{2}}$	0	...
$D^+ \rightarrow K^0 \pi^+$	0	0	0	0	1	0	1	0	1	0	1	0	...
$D^+ \rightarrow K^+ \pi^0$	$\frac{1}{\sqrt{2}}$	0	$\frac{1}{\sqrt{2}}$	0	$-\frac{1}{\sqrt{2}}$	0	$-\frac{1}{\sqrt{2}}$	0	0	0	0	0	...
$D_s \rightarrow K^0 K^+$	1	0	1	1	0	0	0	0	1	0	1	1	...

Equivalence to $SU(3)_F$

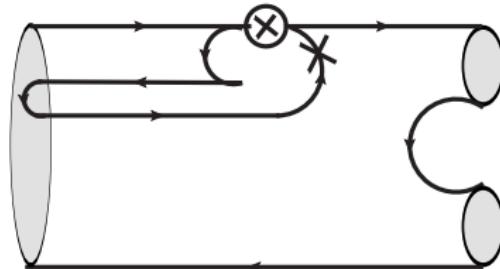
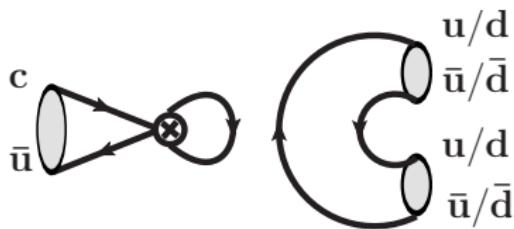
- Diagrammatic parameterization \Leftrightarrow matrix which expresses decay amplitudes in terms of $SU(3)_F$ matrix elements.
- Same rank. Same 6 sum rules. Explicit matching (excerpt):

$SU(3)_F$ ME	...	E	$E_1^{(1)}$	$E_2^{(1)}$	$E_3^{(1)}$	P^{break}
A_{27}^{15}	...	0	0	0	0	0
A_8^{15}	...	$-\frac{5}{2\sqrt{2}}$	$-\frac{5}{3\sqrt{2}}$	$-\frac{5}{6\sqrt{2}}$	0	0
$A_8^{\bar{6}}$...	$\frac{\sqrt{5}}{2}$	0	$\frac{\sqrt{5}}{2}$	0	0
B_1^3	...	0	$\frac{32\sqrt{\frac{35}{421}}}{3}$	$\frac{32\sqrt{\frac{35}{421}}}{3}$	$-\frac{16\sqrt{\frac{35}{421}}}{3}$	$\frac{32\sqrt{\frac{35}{421}}}{3}$
B_8^3	...	0	$-\frac{20\sqrt{\frac{7}{3937}}}{3}$	$-\frac{20\sqrt{\frac{7}{3937}}}{3}$	$\frac{40\sqrt{\frac{7}{3937}}}{3}$	$\frac{160\sqrt{\frac{7}{3937}}}{3}$
$B_8^{\bar{6}_1}$...	0	$20\sqrt{\frac{7}{2869}}$	$-20\sqrt{\frac{7}{2869}}$	0	0
$B_8^{15_1}$...	0	$460\sqrt{\frac{7}{1330969}}$	$20\sqrt{\frac{133}{70051}}$	$-840\sqrt{\frac{7}{1330969}}$	0
$B_8^{15_2}$...	0	$-20\sqrt{\frac{6}{871}}$	$10\sqrt{\frac{6}{871}}$	$10\sqrt{\frac{6}{871}}$	0
$B_{27}^{15_1}$...	0	0	0	0	0
$B_{27}^{15_2}$...	0	0	0	0	0
$B_{27}^{24_1}$...	0	0	0	0	0

Consequences of Equivalence to $SU(3)_F$

- Topological amplitude parameterization is **algebraically complete**.
→ All further diagrammatic contributions can be absorbed.
- Example 1: $PA_{\text{break}} \equiv PA_s - PA_d$ can be absorbed into exchange diagrams.
- Example 2: Contributions from **higher Fock states**:

$$|K^0\rangle = |d\bar{s}\rangle + |d\bar{s}g\rangle + |d\bar{s}u\bar{u}\rangle + \dots .$$



Consistency Constraints on $SU(3)_F$ breaking: Validity of perturbative expansion

Diagrammatic measures of $SU(3)_F$ breaking $\leq 50\%$

① $\delta_X'^{\mathcal{T}} \equiv \max_d |\mathcal{A}_X^{\mathcal{T}}(d)/\mathcal{A}(d)|$, $\mathcal{T} = C, E, P_{\text{break}}$

individual amount of $\cancel{SU(3)_F}$ by topology \mathcal{T} .

② $\delta_X'^{\text{topo}} \equiv \max_d |\sum_{\mathcal{T}} A_X^{\mathcal{T}}(d)/\mathcal{A}(d)|$

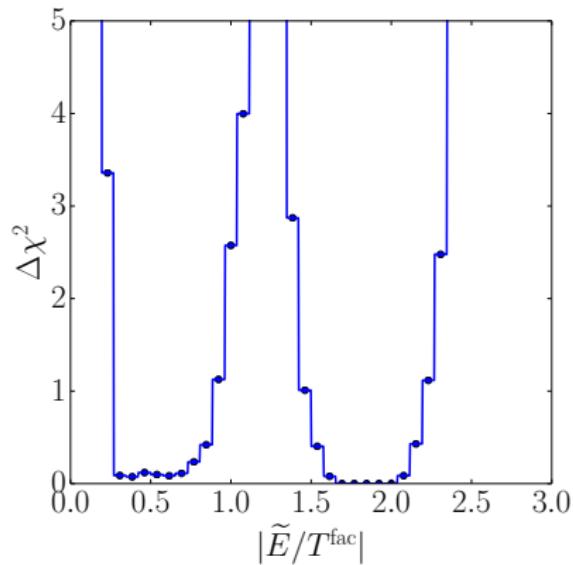
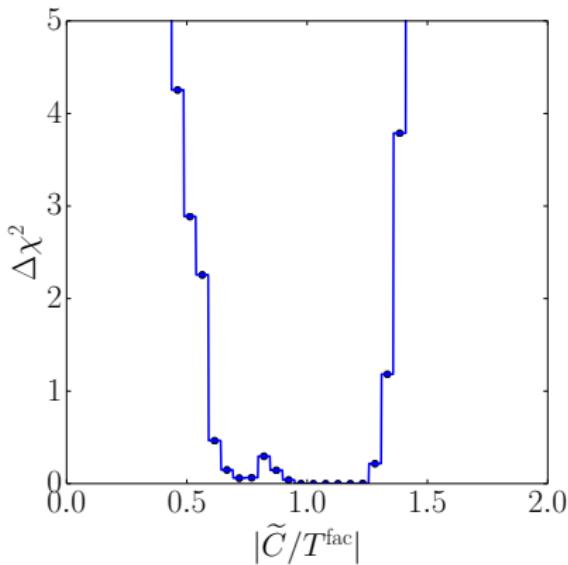
overall amount of $\cancel{SU(3)_F}$.

③ $\delta_X^{C_i/C} \equiv |C_i/C|$ $\cancel{SU(3)_F}$ in C -parameters.

④ $\delta_X^{E_i/E} \equiv |E_i/E|$ $\cancel{SU(3)_F}$ in E -parameters.

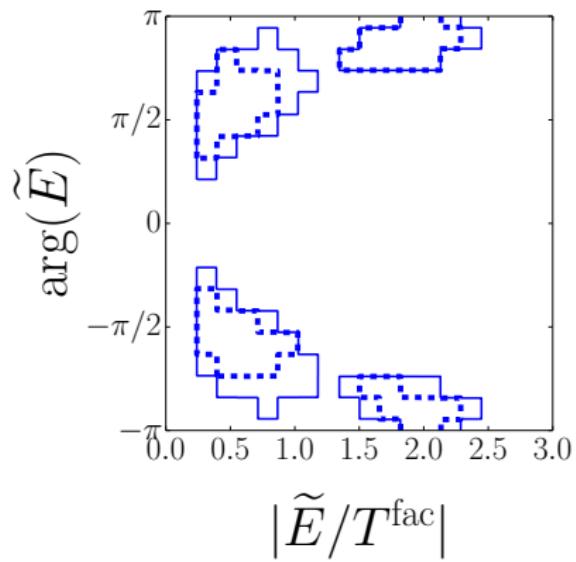
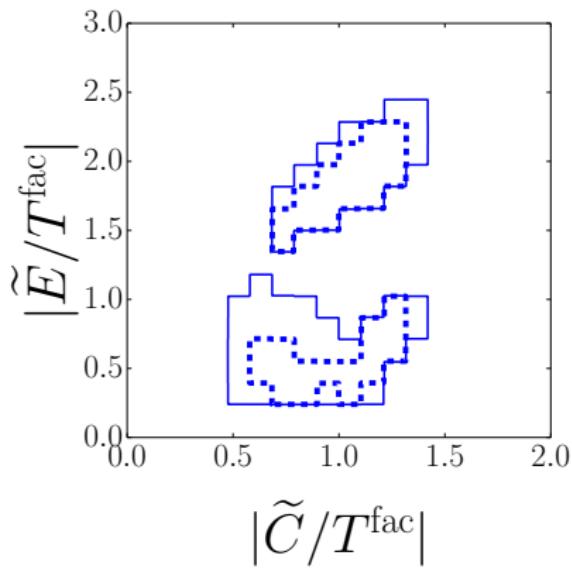
$\mathcal{A}_X^{\mathcal{T}}(d)$: $\cancel{SU(3)_F}$ part of the amplitude of decay d
stemming from topology \mathcal{T} .

Fit results for $SU(3)_F$ -limit parameters



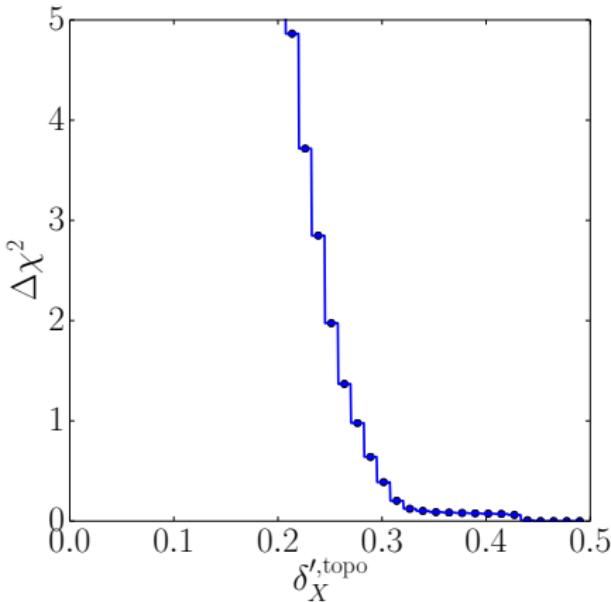
- **Perfect fit** to branching ratios: $\chi^2 \sim 0$: under-determined problem.
But: **Nontrivial** result due to many parameter constraints:
Permit only up to **50%** $SU(3)_F$ -breaking.

Broad and Multiple Fit Solutions



Example: Quantify $SU(3)_F$ -breaking

$\Delta\chi^2$ profile of the parameter $\delta_X'^{\text{topo}}$ which quantifies the overall size of $SU(3)_F$ -breaking:



Results:

- i) The $SU(3)_F$ limit $\delta_X'^{\text{topo}} = 0$ is ruled out by more than 5σ .
- ii) At 68% CL there is at least 28% of $SU(3)_F$ breaking.

Relative Importance of Diagrams: Likelihood Ratio Tests

Hypothesis	Significance of rejection
$P_{\text{break}} = 0$	0.7σ
$P_{\text{break}} = E_i^{(1)} = C_i^{(1)} = 0 \forall i$	$> 5\sigma$
$E_i^{(1)} = 0 \forall i$	3.0σ
$E = E_i^{(1)} = 0 \forall i$	$> 5\sigma$
$C_i^{(1)} = 0 \forall i$	4.3σ
$C = C_i^{(1)} = 0 \forall i$	$> 5\sigma$

- Clear need for $SU(3)_F$ breaking.



- P_{break} allowed to be zero at 0.7σ .

Probe the GIM mechanism in Charm

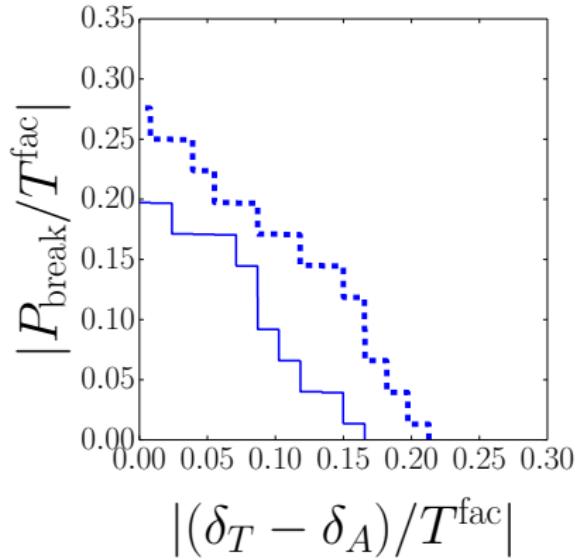


Complementarity of P_{break} and
 $1/N_c^2$ -corrections:

$$\mathcal{B}(D^+ \rightarrow K_{S,L} K^+) = |\lambda_{sd}|^2 \mathcal{P}(D^+, K^0, K^+) \times \\ |\mathcal{A}^{\text{fac}}(D^+ \rightarrow \bar{K}^0 K^+) + (\delta_T - \delta_A) + P_{\text{break}}|^2 ,$$

$$\mathcal{B}(D_s^+ \rightarrow K_{S,L} \pi^+) = |\lambda_{sd}|^2 \mathcal{P}(D_s^+, K^0, \pi^+) \times \\ |\mathcal{A}^{\text{fac}}(D_s^+ \rightarrow K^0 \pi^+) - (\delta_T - \delta_A) + P_{\text{break}}|^2 ,$$

$$\mathcal{B}(D^+ \rightarrow K^+ \pi^0) = |V_{cd}^* V_{us}|^2 \mathcal{P}(D^+, K^+, \pi^0) \times \\ |\mathcal{A}^{\text{fac}}(D^+ \rightarrow K^+ \pi^0) + (\delta_T - \delta_A)|^2 ,$$



Probe the quality of the $1/N_c$ expansion

Sum rules between $D^+ \rightarrow K_S K^+$, $D_s^+ \rightarrow K_S \pi^+$ and $D^+ \rightarrow K^+ \pi^0$

$$\tilde{\mathcal{A}}(D^+ \rightarrow \bar{K}^0 K^+) - \tilde{\mathcal{A}}(D_s^+ \rightarrow K^0 \pi^+) = 2(\delta_T - \delta_A)$$

$$\tilde{\mathcal{A}}(D^+ \rightarrow K^+ \pi^0) = \frac{1}{\sqrt{2}}(\delta_T - \delta_A)$$

Combination of both:

$$\tilde{\mathcal{A}}(D^+ \rightarrow \bar{K}^0 K^+) - \tilde{\mathcal{A}}(D_s^+ \rightarrow K^0 \pi^+) - 2\sqrt{2}\tilde{\mathcal{A}}(D^+ \rightarrow K^+ \pi^0) = 0$$

Definition

$$\tilde{\mathcal{A}}(d) \equiv \mathcal{A}(d) - \mathcal{A}^{\text{fac}}(d),$$

$$\mathcal{A}^{\text{fac}}(d) \equiv T^{\text{fac}}(d) + A^{\text{fac}}(d).$$

Predictions for Branching Ratios

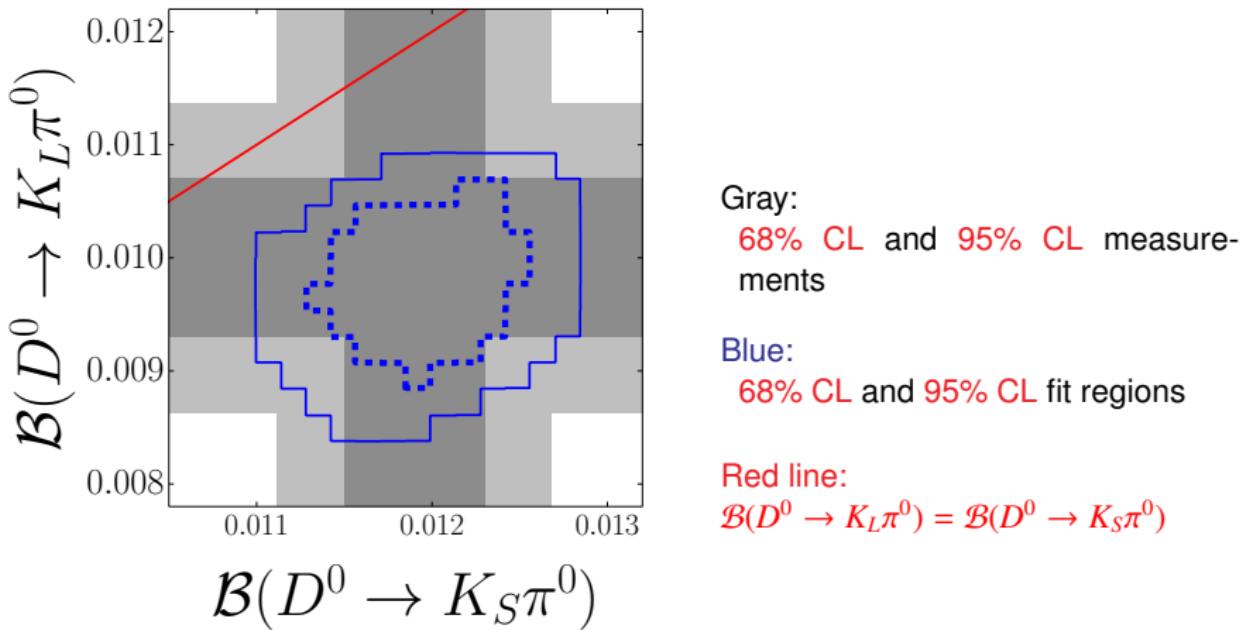
- In general, individual branching ratios predicted from global fit not more precise than current measurements.
- Exception: Probe DCS amplitudes.
- $A(D^0 \rightarrow \bar{K}^0 \pi^0)$ (CF) and $A(D^0 \rightarrow K^0 \pi^0)$ (DCS) interfere.
 $\Rightarrow A(D^0 \rightarrow K_S \pi^0)$ and $A(D^0 \rightarrow K_L \pi^0)$
- $SU(3)_F$ limit:

$$\mathcal{B}(D^0 \rightarrow K_S \pi^0) \propto |E - C|^2 + 2\lambda^2 |E - C|^2$$

$$\mathcal{B}(D^0 \rightarrow K_L \pi^0) \propto |E - C|^2 - 2\lambda^2 |E - C|^2$$

Can $SU(3)_F$ breaking change the $SU(3)_F$ -limit prediction
 $\mathcal{B}(D^0 \rightarrow K_L \pi^0) < \mathcal{B}(D^0 \rightarrow K_S \pi^0)$?

Probe of DCS amplitudes I



While $SU(3)_F$ breaking can be sizable, $\mathcal{B}(D^0 \rightarrow K_L\pi^0) < \mathcal{B}(D^0 \rightarrow K_S\pi^0)$ holds with a significance of more than 4σ .

Probe of DCS amplitudes II

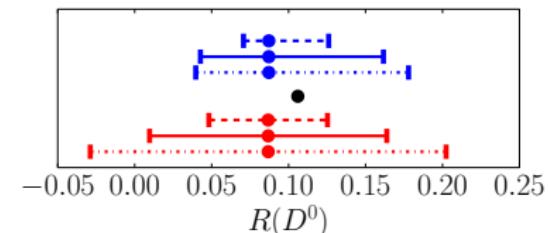
Formulated differently

Asymmetry

$$R(D^0) \equiv \frac{\mathcal{B}(D^0 \rightarrow K_S\pi^0) - \mathcal{B}(D^0 \rightarrow K_L\pi^0)}{\mathcal{B}(D^0 \rightarrow K_S\pi^0) + \mathcal{B}(D^0 \rightarrow K_L\pi^0)}$$

Blue: $1, 2, 3\sigma$. Black: $SU(3)_F$ -limit.

[Bigi Yamamoto 1994, Rosner 2006,
Gao 2006]



$\mathcal{B}(D_s^+ \rightarrow K_L K^+)$ not measured yet.

Prediction:

$$\mathcal{B}(D_s^+ \rightarrow K_L K^+) = 0.012^{+0.007}_{-0.002} \text{ at } 3\sigma$$

$$R(D_s^+) \equiv \frac{\mathcal{B}(D_s^+ \rightarrow K_S K^+) - \mathcal{B}(D_s^+ \rightarrow K_L K^+)}{\mathcal{B}(D_s^+ \rightarrow K_S K^+) + \mathcal{B}(D_s^+ \rightarrow K_L K^+)}$$

Black: QCDF@ 1σ [Gao 2014]

CP Asymmetries: Theory service for Experiment

- Every measurement hinting for $a_{CP}^{\text{dir}} \neq 0$ was successfully postdicted...
 - ...in the Standard Model.
 - ...or New Physics Models.



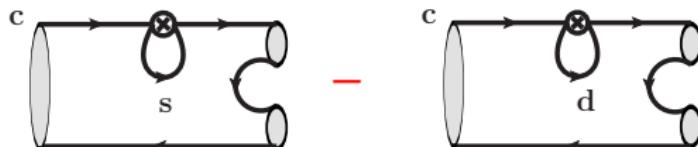
- **Why is that?**



Problem of CP Asymmetry Predictions:

- New hadronic quantities appear which cannot be extracted from \mathcal{B} measurements.

- \mathcal{B} 's involve only



⇒ Difference can be extracted.



- A_{CP} 's involve also



The sum is unknown.

[Brod, Grossman, Kagan, Zupan 2012]

Solution: CP asymmetry sum rules

Strategy: Sum rules among CP asymmetries.

- Build combinations out of several CP asymmetries...
- ... containing only those topological amplitudes in coefficients which can be extracted from the global fit to the branching ratios.

Extent known $SU(3)_F$ limit sum rules

[see, e.g., Grossman Kagan Nir 2006, Hiller Jung Schacht 2012, Grossman Ligeti Robinson 2014]

$$a_{CP}^{\text{dir}}(D^0 \rightarrow K^+ K^-) + a_{CP}^{\text{dir}}(D^0 \rightarrow \pi^+ \pi^-) = 0,$$

$$a_{CP}^{\text{dir}}(D^+ \rightarrow \bar{K}^0 K^+) + a_{CP}^{\text{dir}}(D_s^+ \rightarrow K^0 \pi^+) = 0,$$

valid at zeroth order $SU(3)_F$ breaking.

- Include corrections of sum rules due to $SU(3)_F$ breaking in the CKM-leading part of the amplitude...

Result

Two sum rules each correlating **three** direct CP asymmetries

I $D^0 \rightarrow K^+K^-$, $D^0 \rightarrow \pi^+\pi^-$, and $D^0 \rightarrow \pi^0\pi^0$,
and

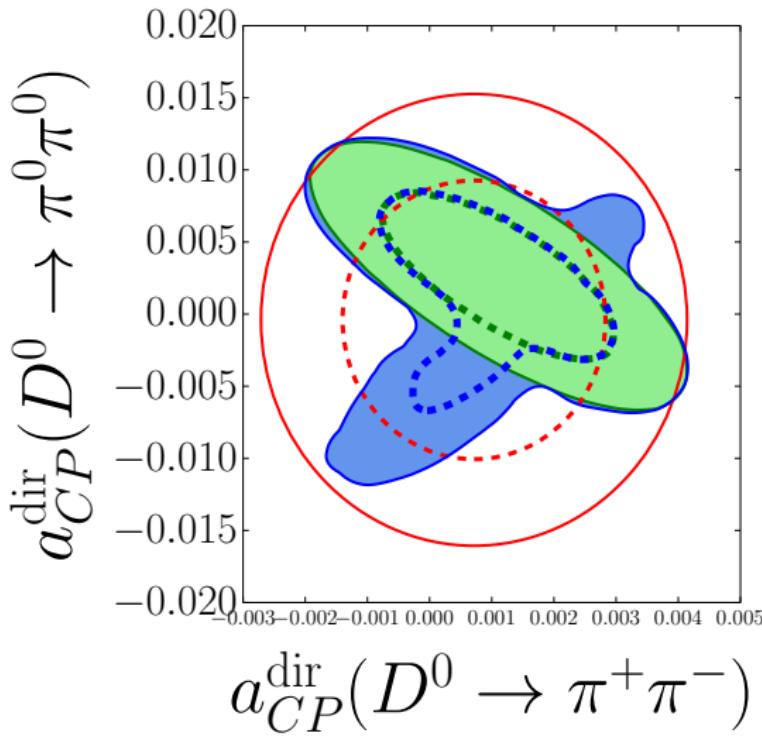
II $D^+ \rightarrow \bar{K}^0K^+$, $D_s^+ \rightarrow K^0\pi^+$, and $D_s^+ \rightarrow K^+\pi^0$.

- Note: Still works to **zeroth** order in $SU(3)_F$ breaking **only**,
as $SU(3)_F$ breaking in **CKM-subleading** part of amplitudes is **not**
taken into account, e.g. $SU(3)_F$ breaking of $P_s + P_d$.
- Still: theoretical accuracy of **new-physics tests** only limited by the
assumed size of $SU(3)_F$ breaking, i.e. generically $O(30\%)$.
- **Great progress** compared to $O(1000\%)$ spread of past predictions.

➡ Look at phenomenological implications.

Implications of sum rule I

[preliminary]



Red solid:

95% CL measurement

Red dashed:

68% CL measurement

Present data:

Light blue:

95% CL from global fit

Dark blue dashed:

68% CL from global fit

Future scenario:

assume $\sqrt{50}$ better
branching ratios, but
 $a_{CP}^{dir}(D^0 \rightarrow K^+K^-)$ as to-
day.

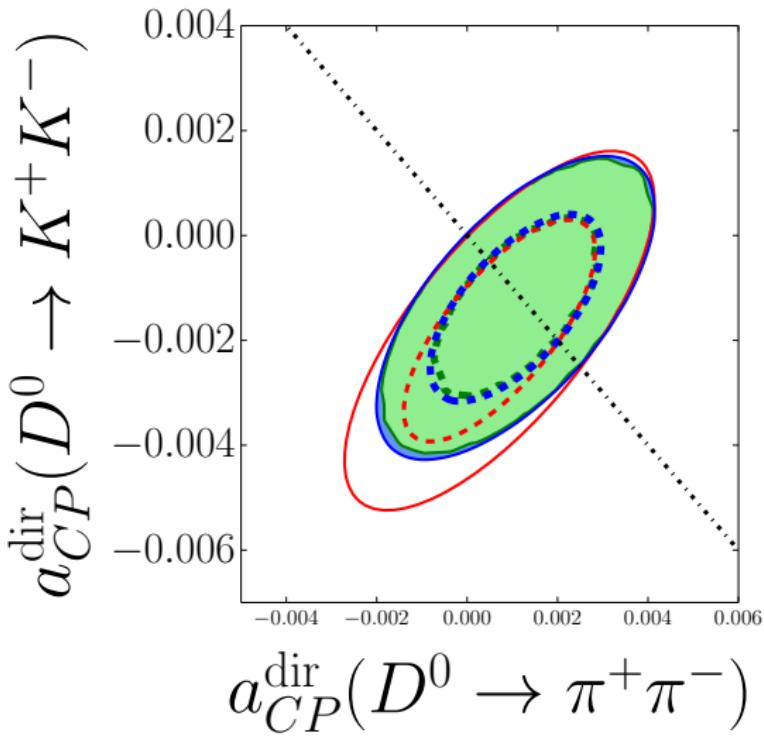
Light green:

95% CL from global fit

Dark green dashed:

68% CL from global fit

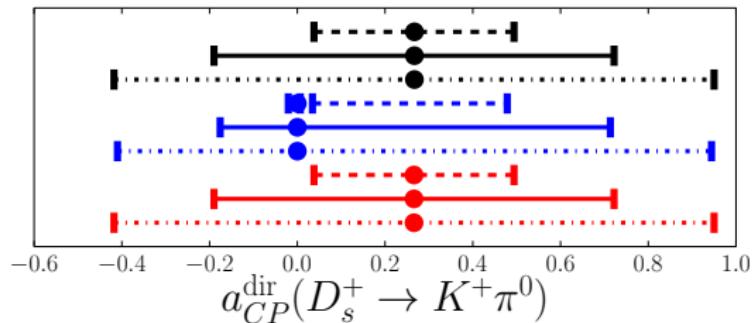
Implications of sum rule I, contd. [preliminary]



Implications of sum rule II [preliminary]

Use measured values of $D^+ \rightarrow \bar{K}^0 K^+$ and $D_s^+ \rightarrow K^0 \pi^+$ to predict

$a_{CP}^{\text{dir}}(D_s^+ \rightarrow K^+ \pi^0)$:



Blue: prediction from $a_{CP}^{\text{dir}}(D^+ \rightarrow \bar{K}^0 K^+)$, $a_{CP}^{\text{dir}}(D_s^+ \rightarrow K^0 \pi^+)$, and global fit to branching ratios.

Black: same as blue, but without $1/N_c$ constraints.

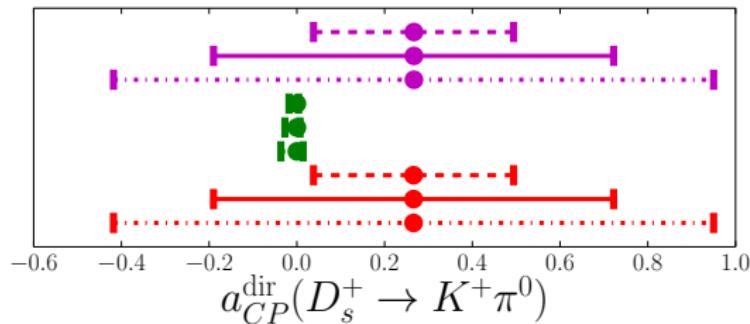
Red: measurement. Dashed: 1σ , solid: 2σ , dot-dashed: 3σ .

Not shown: error from $SU(3)_F$ breaking in $P_s + P_d$.

⇒ yet another successful postdiction.

Implications of sum rule II, future scenario [preliminary]

But: Assuming better measurements of the branching ratios by a factor of $\sqrt{50}$ changes the picture:



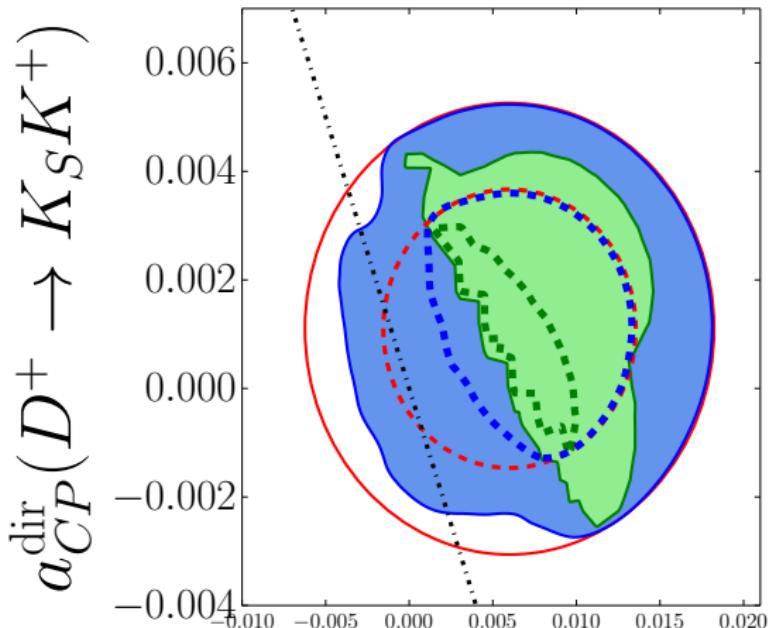
Green: prediction from $a_{CP}^{\text{dir}}(D^+ \rightarrow \bar{K}^0 K^+)$, $a_{CP}^{\text{dir}}(D_s^+ \rightarrow K^0 \pi^+)$, and global fit to branching ratios.

Magenta: same as blue, but without $1/N_c$ constraints.

Red: measurement. Dotted: 1σ , solid: 2σ , dot-dashed: 3σ .

Not shown: error from $SU(3)_F$ breaking in $P_s + P_d$.

Implications of sum rule II, fixed $a_{CP}^{\text{dir}}(D_s^+ \rightarrow K^+\pi^0) = -1\%$



$$a_{CP}^{\text{dir}}(D_s^+ \rightarrow K_S \pi^+)$$

Black dashed: $SU(3)_F$ limit

Red solid:

95% CL measurement

Red dashed:

68% CL measurement

Present data:

Light blue:

95% CL from global fit

Dark blue dashed:

68% CL from global fit

Future scenario:

assume $\sqrt{50}$ better branching ratios.

Light green:

95% CL from global fit

Dark green dashed:

68% CL from global fit

Conclusion

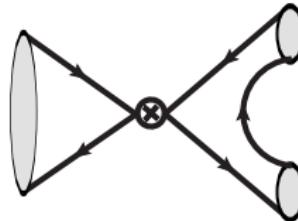
- Global fit of $D \rightarrow PP'$ branching ratios to topological amplitudes including linear $SU(3)_F$ breaking and $1/N_c$ -counting gives multiple degenerate best-fit solutions.
- The method permits likelihood ratio test to quantify e.g. the size of $SU(3)_F$ breaking and $P_{\text{break}} \neq 0$ (GIM).
- We predict:
$$\mathcal{B}(D_s^+ \rightarrow K_L K^+) = 0.012^{+0.007}_{-0.002} \quad \text{at } 3\sigma$$
$$\mathcal{B}(D^0 \rightarrow K_L \pi^0) < \mathcal{B}(D^0 \rightarrow K_S \pi^0) \quad \text{at } 4\sigma$$
- CP asymmetries involve topological amplitudes not constrained by the fit. These can be eliminated by forming judicious combinations of several CP asymmetries → sum rules .
- The sum rules test the quality of $SU(3)_F$ in penguin amplitudes and/or new physics.

Wish list for Lattice QCD

- Any information just on a single $SU(3)_F$ limit topological amplitude would be invaluable.
- Desperately needed:

Scalar $K \rightarrow \pi$ form factor $F_0^{K\pi}(m_{D_{(s)}}^2)$ (i.e. @high $q^2 \sim 3.4 \text{ GeV}^2$).

- We use $1 \lesssim |F_0^{K\pi}(m_{D_{(s)}}^2)| \lesssim 4.5$ and $-\pi \lesssim \arg(F_0^{K\pi}(m_{D_{(s)}}^2)) \lesssim \pi$ to accommodate poor information from $\tau \rightarrow K_S \pi^- \nu_\tau$ data [Belle, 0706.2231].
- ↳ Much better control of annihilation diagrams would be possible.



- ↳ Test of $1/N_c$ expansion with $\mathcal{B}(D^+ \rightarrow K^+ \pi^0) \sim |\mathcal{A}^{\text{fac}}(D^+ \rightarrow K^+ \pi^0) + (\delta_T - \delta_A)|^2$ would be feasible.
- ↳ Improved test of GIM in charm would be possible, e.g. with $\mathcal{B}(D^+ \rightarrow K_{S,L} K^+) \sim |\mathcal{A}^{\text{fac}}(D^+ \rightarrow \bar{K}^0 K^+) + (\delta_T - \delta_A) + P_{\text{break}}|^2$.